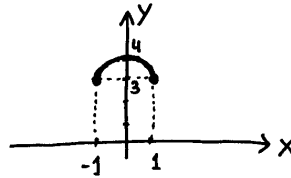
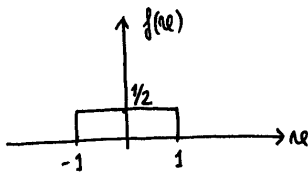


FEUP - LEEC - Probabilidades e Estatística

Exercícios Resolvidos - Folha 3

1.



1º PASSO) Determinar a distribuição acumulada de Y .

$$\begin{aligned}
 G(y) &= P(Y \leq y) \\
 &= P(4 - X^2 \leq y) \\
 &= P(X^2 \geq 4 - y) \\
 &= P(X \geq \sqrt{4-y} \cup X \leq -\sqrt{4-y}) \quad \text{Acontecimentos mutuamente exclusivos} \\
 &= P(X \geq \sqrt{4-y}) + P(X \leq -\sqrt{4-y}) =
 \end{aligned}$$

Se $0 \leq \sqrt{4-y} \leq 1$:

$$\int_{\sqrt{4-y}}^1 \frac{1}{2} dx = \frac{1}{2} (1 - \sqrt{4-y})$$

Se $-1 \leq -\sqrt{4-y} < 0$:

$$\int_{-1}^{-\sqrt{4-y}} \frac{1}{2} dx = \frac{1}{2} (1 - \sqrt{4-y})$$

Se $\sqrt{4-y} > 1$:

$$\int_{\sqrt{4-y}}^{+\infty} 0 dx = 0$$

Se $-\sqrt{4-y} < -1$:

$$\int_{-\infty}^{-\sqrt{4-y}} 0 dx = 0$$

Como:

$$\begin{aligned}
 0 < \sqrt{4-y} < 1 &\Leftrightarrow 3 < y < 4 \\
 -1 < -\sqrt{4-y} < 0 &\Leftrightarrow 1 > \sqrt{4-y} > 0 \Leftrightarrow 3 < y < 4 \\
 \sqrt{4-y} > 1 &\Leftrightarrow y < 3 \\
 -\sqrt{4-y} < -1 &\Leftrightarrow \sqrt{4-y} > 1 \Leftrightarrow y < 3
 \end{aligned}$$

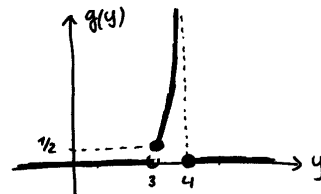
Vem para $G(y)$:

$$G(y) = \begin{cases} 0, & y < 3 \\ 1 - \sqrt{4-y}, & 3 \leq y < 4 \\ 1, & y \geq 4 \end{cases}$$

↳ por definição de distribuição acumulada.

Logo, como $g(y) = \frac{dG(y)}{dy}$:

$$g(y) = \begin{cases} 0, & y < 3 \\ \frac{1}{2\sqrt{4-y}}, & 3 \leq y < 4 \\ 0, & y \geq 4 \end{cases}$$



2. $X = \{\text{número de vezes que sai 6 no lançamento de 4 dados}\}$

i	x_i	$p(x_i)$
1	0	$\frac{625}{1296}$
2	1	$\frac{500}{1296}$
3	2	$\frac{150}{1296}$
4	3	$\frac{20}{1296}$
5	4	$\frac{1}{1296}$

$$E(X) = \sum_{i=1}^5 (x_i \times p(x_i)) = 0 \times \frac{625}{1296} + 1 \times \frac{500}{1296} + 2 \times \frac{150}{1296} + 3 \times \frac{20}{1296} + 4 \times \frac{1}{1296} = \frac{2}{3}$$

$$E(X^2) = \sum_{i=1}^5 (x_i^2 \times p(x_i)) = 0^2 \times \frac{625}{1296} + 1^2 \times \frac{500}{1296} + 2^2 \times \frac{150}{1296} + 3^2 \times \frac{20}{1296} + 4^2 \times \frac{1}{1296} = 1$$

$$V(X) = E(X^2) - E^2(X) = 1 - \left(\frac{2}{3}\right)^2 = \frac{5}{9}$$

3.

$$f(x) = 6x(1-x), \quad 0 \leq x \leq 1$$

$$E(X) = \int_{-\infty}^{+\infty} x f(x) dx = \int_0^1 x \cdot 6x(1-x) dx = \int_0^1 (6x^2 - 6x^3) dx = \left[\frac{6x^3}{3} - \frac{6x^4}{4} \right]_0^1 =$$

$$= \frac{6}{3} - \frac{6}{4} = \frac{1}{2}$$

$$E(X^2) = \int_{-\infty}^{+\infty} x^2 f(x) dx = \int_0^1 x^2 \cdot 6x(1-x) dx = \dots = \frac{3}{10}$$

$$V(X) = E(X^2) - [E(X)]^2 = \frac{3}{10} - \left(\frac{1}{2}\right)^2 = \frac{1}{20}$$

4.

2)

 X_n - PONTUAÇÃO OBTIDA NUMA PERGUNTA

$$i) E(X_n) = \sum_{i=1}^{\infty} u_{n,i} \cdot p_{X_n}(u_{n,i})$$

$$E(X_0) = 0 \times 1 = 0$$

$$E(X_1) = 3 \times \frac{1}{3} - 1 \times \frac{2}{3} = 0$$

$$E(X_2) = 2 \times \frac{1}{2} - 2 \times \frac{1}{2} = 0$$

$$E(X_3) = 1 \times \frac{2}{4} - 3 \times \frac{1}{4} = 0$$

$$E(X_4) = 0 \times 1 = 0$$

$$ii) E(X_n^2) = \sum_{i=1}^{\infty} u_{n,i}^2 \cdot p_{X_n}(u_{n,i})$$

$$E(X_0^2) = 0^2 \times 1 = 0$$

$$E(X_1^2) = 3^2 \times \frac{1}{3} + 1^2 \times \frac{2}{3} = 3$$

$$E(X_2^2) = 2^2 \times \frac{1}{2} + 2^2 \times \frac{1}{2} = 4$$

$$E(X_3^2) = 1^2 \times \frac{2}{4} + 3^2 \times \frac{1}{4} = 3$$

$$E(X_4^2) = 0^2 \times 1 = 0$$

$$b) i) S_n = \sum_{i=1}^{34} X_i$$

$$E(S_1) = \sum_{k=1}^{34} E(X_1) = 34 \times 0 = 0 \quad V(S_1) = V(34 \cdot X_1) = 34^2 \cdot V(X_1) = 1156 \times 3 = 3468 \Rightarrow \sigma_{S_1} = 58.9$$

$$E(S_2) = 34 \times 0 = 0 \quad V(S_2) = V(34 \cdot X_2) = 34^2 \cdot V(X_2) = 4624 \Rightarrow \sigma_{S_2} = 68.0$$

$$E(S_3) = 34 \times 0 = 0 \quad V(S_3) = V(34 \cdot X_3) = 34^2 \cdot V(X_3) = 3468 \Rightarrow \sigma_{S_3} = 58.9$$

	X_0	X_1	X_2	X_3	X_4
n	0	1	2	3	4
CERTO ERRADO	—	C	C	CC	CCC
X_n	0	3	-1	2	-2
p	1	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{3}{4}$

$$\frac{\binom{1}{1} \binom{3}{1}}{\binom{4}{2}} = \frac{1 \times 3}{6} = \frac{1}{2}$$

$$\frac{\binom{1}{1} \binom{3}{2}}{\binom{4}{3}} = \frac{1 \times 3}{4} = \frac{3}{4}$$

$$\frac{\binom{3}{3}}{\binom{4}{4}} = \frac{1}{4}$$

$$V(X_n) = E(X_n^2) - E^2(X_n)$$

$$V(X_0) = 0 - 0^2 = 0$$

$$V(X_1) = 3 - 0^2 = 3$$

$$V(X_2) = 4 - 0^2 = 4$$

$$V(X_3) = 3 - 0^2 = 3$$

$$V(X_4) = 0 - 0^2 = 0$$

$$1a) P(|X-\mu| \geq c\sigma) \leq \frac{1}{c^2} \text{ and } P(|X-\mu| \geq \delta) \leq \frac{\sigma^2}{\delta^2}$$

$$P(S_1 \geq 20) = \frac{1}{2} \cdot P(|S_1| \geq 20) = \frac{1}{2} \cdot P(|S_1 - 0| \geq 20 - 0) \leq \frac{1}{2} \cdot \frac{\sigma_{S_1}^2}{20^2} = 4.335$$

$\uparrow$
 média de S_1

$$P(S_1 \geq 100) = \frac{1}{2} (P(|S_1| \geq 100)) = \dots \leq \frac{1}{2} \cdot \frac{\sigma_{S_1}^2}{100^2} = 0.1734$$

$$P(S_2 \geq 20) = \frac{1}{2} (P(|S_2| \geq 20)) = \dots \leq \frac{1}{2} \cdot \frac{\sigma_{S_2}^2}{20^2} = 5.78$$

$$P(S_2 \geq 100) = \frac{1}{2} (P(|S_2| \geq 100)) = \dots \leq \frac{1}{2} \cdot \frac{\sigma_{S_2}^2}{100^2} = 0.2312$$

$$P(S_3 \geq 20) = \frac{1}{2} (P(|S_3| \geq 20)) = \dots \leq \frac{1}{2} \cdot \frac{\sigma_{S_3}^2}{20^2} = 4.335$$

$$P(S_3 \geq 100) = \frac{1}{2} (P(|S_3| \geq 100)) = \dots \leq \frac{1}{2} \cdot \frac{\sigma_{S_3}^2}{100^2} = 0.1735$$

COMENTÁRIOS: FAZENDO O TESTE À SORTE, É PROVÁVEL OBTER MAIS DO QUE 20 PONTOS COM 1, 2 OU 3 CRUZES POR PERGUNTA. JÁ A PROBABILIDADE DE OBTER MAIS DO QUE 100 PONTOS, É MUITO PEQUENA (NESTE CASO, ARRISCAR EM DUAS CRUZES PODEM TRAZER BENEFÍCIOS).

c) RESOLUÇÃO ANALOGA A ALÍNEA a)

5. ...

6. ...

7. ...

8. ...