

FEUP - LEEC - Probabilidades e Estatística

Exercícios Resolvidos - Folha 4

1.

RESOLUÇÃO :

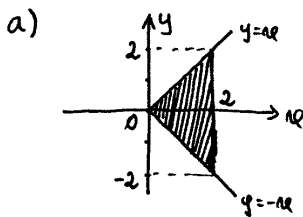
$p(x_i, y_j)$		X		
		0	1	2
Y	0	5/36	0	1/36
	1	3/36	2/36	0
	2	6/36	2/36	0
	3	4/36	2/36	0
	4	2/36	2/36	0
	5	0	2/36	0

X	0	1	2
$g(x_i) = \sum_{j=0}^5 p(x_i, y_j)$	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

Y	$h(y_j) = \sum_{i=0}^5 p(x_i, y_j)$
0	6/36
1	10/36
2	8/36
3	6/36
4	4/36
5	2/36

2.

RESOLUÇÃO :



$$\int_0^2 \int_{-x}^x k x(x-y) dy dx = 1 \quad (\Leftarrow)$$

(...)

$$8k = 1 \quad (\Leftarrow)$$

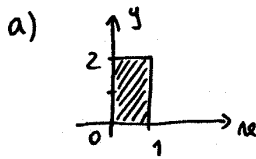
$$k = \frac{1}{8}$$

$$b) \quad f_x(x) = \int_{-x}^x \frac{1}{8} x(x-y) dy = \dots = \frac{x^3}{4}, \quad 0 < x < 2$$

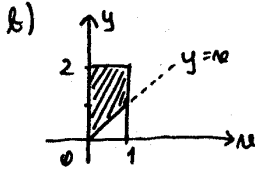
$$f_y(y) = \begin{cases} \int_{-y}^2 \frac{1}{8} x(x-y) dx = \dots = \frac{1}{3} - \frac{y}{4} + \frac{5y^3}{48}, & -2 < y < 0 \\ \int_y^2 \frac{1}{8} x(x-y) dx = \dots = \frac{1}{3} - \frac{y}{4} + \frac{y^3}{48}, & 0 < y < 2 \end{cases}$$

3.

RESOLUCIÓN:



$$\int_{1/2}^1 \int_0^2 (x^2 + \frac{xy}{3}) dy dx = \dots = \frac{5}{6}$$



$$P(Y > X) = \int_0^1 \int_x^2 (x^2 + \frac{xy}{3}) dy dx = \dots = \frac{17}{24}$$

c)

$$P(Y < \frac{1}{2} | X < \frac{1}{2}) = \frac{P(Y < \frac{1}{2} \wedge X < \frac{1}{2})}{P(X < \frac{1}{2})} = \frac{\int_0^{1/2} \int_0^{1/2} (x^2 + \frac{xy}{3}) dy dx}{\frac{1}{6}} = \dots = \frac{5}{32}$$

$\hookrightarrow$ ver alínea a)

d)

$$\rho_{xy} = \frac{E(XY) - E(X)E(Y)}{\sigma_x \sigma_y}$$

Calcular auxiliares:

$$f_x(x) = \int_0^2 (x^2 + \frac{xy}{3}) dy = 2x^2 + \frac{2}{3}x$$

$$f_y(y) = \int_0^1 (x^2 + \frac{xy}{3}) dx = \frac{1}{3} + \frac{1}{6}y$$

$$E(XY) = \int_0^1 \int_0^2 \overbrace{xy (x^2 + \frac{xy}{3})}^{f(x,y)} dy dx = \dots = \frac{43}{54}$$

$$E(X) = \int_0^1 x f_x(x) dx = \int_0^1 x (2x^2 + \frac{2}{3}x) dx = \dots = \frac{13}{18}$$

$$E(Y) = \int_0^2 y f_y(y) dy = \int_0^2 y (\frac{1}{3} + \frac{1}{6}y) dy = \dots = \frac{10}{9}$$

$$\sigma_x^2 = V(X) = E(X^2) - [E(X)]^2 = \frac{73}{1620}$$

$$\hookrightarrow \int_0^1 \underbrace{x^2 (2x^2 + \frac{2}{3}x)}_{f_x(x)} dx = \dots = \frac{13}{30}$$

$$\sigma_y^2 = V(Y) = E(Y^2) - [E(Y)]^2 = \frac{26}{81}$$

$$\hookrightarrow \int_0^2 y^2 \underbrace{\left(\frac{1}{3} + \frac{1}{6}y\right)}_{f_Y(y)} dy = \dots = \frac{26}{81}$$

$$\rho_{xy} = \frac{\frac{43}{54} - \frac{13}{18} \times \frac{10}{9}}{\sqrt{\frac{73}{1620}} \times \sqrt{\frac{26}{81}}} \simeq -0.051$$

Logo X e Y estão muito fracamente (matematicamente não estão) correlacionadas.

4. ...

5. ...